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ANALYTICAL CALCULATION OF THE AREAS
OF SATURN'S DISK AND RINGS

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February 1977

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#### ANALYTICAL CALCULATION OF THE AREAS

### OF SATURN'S DISK AND RINGS

#### S. Matthews and E. F. Erickson

Ames Research Center

#### SUMMARY

Determination of the thermal emission from the disk of Saturn at wavelengths in the vicinity of the thermal peak (-50  $\mu m$ ) is complicated by the fact that infrared telescopes currently operating at these wavelengths cannot spatially separate the rings from the disk. To account for the emission from the rings, the area of the visible disk, the area of the ansae (visible rings not overlapping the disk), and the area of overlap (visible rings overlapping the disk) must be known. The calculation presented here describes the analytical determination of these areas from parameters available in the ephemeris.

Saturn's rings and disk make comparable contributions to the thermally emitted flux from the entire system. If one assumes that the optical depth of the rings is not necessarily high, and that the only significant ring effects are due to the A and B rings, then five areas are important in estimating the relative contributions of the rings and disk. These are:

 $\Omega_{\rm vd}$  = unobscured (visible) area of the disk

 $\Omega_A, \Omega_B$  = visible area of the A and B rings respectively

 $\omega_A, \omega_B$  = area of disk obscured by the A and B rings respectively

The brightness of the entire system is described by brightness temperature T, which is related to the rings' temperatures,  $T_A$  and  $T_B$ , and the disk temperature,  $T_d$ , by

$$\begin{split} B_{\nu}(T)\Omega_{\mathsf{t}} &= B_{\nu}(T_{\mathsf{d}})(\Omega_{\mathsf{vd}} + \mathrm{e}^{-\tau A}\omega_{\mathsf{A}} + \mathrm{e}^{-\tau B}\omega_{\mathsf{B}}) \\ &+ B_{\nu}(T_{\mathsf{A}})(1 - \mathrm{e}^{-\tau A})\Omega_{\mathsf{A}} + B_{\nu}(T_{\mathsf{B}})(1 - \mathrm{e}^{-\tau B})\Omega_{\mathsf{B}} \end{split}$$

Here By is the Planck function,

$$\Omega_{\mathsf{t}} = \Omega_{\mathsf{vd}} + \Omega_{\mathsf{A}} + \Omega_{\mathsf{B}} \tag{1}$$

is the total system area, and  $\tau_{\mbox{\scriptsize A}}$  and  $\tau_{\mbox{\scriptsize B}}$  are the optical depths of the rings.

Both the disk and rings are elliptical in appearance to the observer. In rectangular coordinates the equation of an illipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 , \quad a \ge b$$
 (2)

If we transform this to polar coordinates we have  $x = r \cos \theta$ ,  $y = r \sin \theta$ , so

$$r^2 \left( \frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2} \right) = 1$$
 or  $r(\theta) = ab(a^2 \sin^2 \theta + b^2 \cos^2 \theta)^{-1/2}$  (3)

The area of an ellipse is  $\pi ab$ . Our notation for the Saturn geometry is shown in figure 1. Thus, for the problem at hand, we have

$$\Omega_{\rm vd} = \pi a_{\rm o} b_{\rm o} - \omega_{\rm A} - \omega_{\rm B} \tag{4}$$

$$\Omega_{A} = \pi(a_{1}b_{1} - a_{2}b_{2}) - \omega_{A}$$

$$\Omega_{B} = \pi(a_{3}b_{3} - a_{4}b_{4}) - \omega_{B}$$
(5)

If we note that (see ref. 1)

$$\frac{a_2}{a_1} = \frac{b_2}{b_1} = 0.8801 \tag{6}$$

$$\frac{a_3}{a_1} = \frac{b_3}{b_1} = 0.8599 \tag{7}$$

and

$$\frac{a_4}{a_1} = \frac{b_4}{b_1} = 0.6650 \tag{8}$$

then the quantities  $a_2$ ,  $b_2$ ,  $a_3$ ,  $b_3$ ,  $a_4$ , and  $b_4$  can be eliminated in (5) so that

$$\Omega_{A} = 0.2254 \pi a_{1} b_{1} - \omega_{A}$$

$$\Omega_{B} = 0.2972 \pi a_{1} b_{1} - \omega_{B}$$
(9)

ao, bo, al, and bl can be obtained from the ephemeris.

There remains simply to calculate  $~\omega_A$  and  $\omega_B^{}.~$  To determine the angles  $\theta_{\,\bf i}^{}$  , i = 1 to 4, we note that

$$r_{i}(\theta_{i}) = r_{o}(\theta_{i}) \text{ or } r_{i}^{2}(\theta_{i}) = r_{o}^{2}(\theta_{i})$$
 (10)

Then

$$\frac{a_{i}^{2}b_{i}^{2}}{a_{i}^{2}\sin^{2}\theta_{i} + b_{i}^{2}\cos^{2}\theta_{i}} = \frac{a_{o}^{2}b_{o}^{2}}{a_{o}^{2}\sin^{2}\theta_{i} + b_{o}^{2}\cos^{2}\theta_{i}}$$
(11)

which can be solved for tan  $\theta_i$ 

$$t_{i} = \tan \theta_{i} = \frac{b_{0}b_{i}}{a_{0}a_{i}} \left(\frac{a_{i}^{2} - a_{0}^{2}}{b_{0}^{2} - b_{i}^{2}}\right)^{1/2}$$
 for  $b_{i} \leq b_{0}$  (12)

If  $b_i$  should be greater than  $b_0$ ,  $t_i = +\infty$  [ $\theta_i = (\pi/2)$ ]. Now

$$\omega_{A} = 2 \int_{\theta_{2}}^{\theta_{1}} \int_{r_{2}}^{r_{0}} r \, dr \, d\theta + 2 \int_{\theta_{1}}^{\pi/2} \int_{r_{2}}^{r_{1}} r \, dr \, d\theta$$
 (13)

$$= \int_{\theta_2}^{\theta_1} (r_0^2 - r_2^2) d\theta + \int_{\theta_1}^{\pi/2} (r_1^2 - r_2^2) d\theta$$
 (14)

$$= \int_{\theta_2}^{\theta_1} r_0^2 d\theta + \int_{\theta_1}^{\pi/2} r_1^2 d\theta - \int_{\theta_2}^{\pi/2} r_2^2 d\theta$$
 (15)

$$= a_0 b_0 \tan^{-1} \left( \frac{a_0}{b_0} \tan \theta \right) \Big|_{\theta_2}^{\theta_1} + a_1 b_1 \tan^{-1} \left( \frac{a_1}{b_1} \tan \theta \right) \Big|_{\theta_1}^{\pi/2}$$

$$-a_2b_2 \tan^{-1}\left(\frac{a_2}{b_2} \tan \theta\right)\Big|_{\theta_2}^{\pi/2} \tag{16}$$

since

$$\int r^2 d\theta = a^2b^2 \int \frac{d\theta}{a^2 \sin^2 \theta + b^2 \cos^2 \theta} = ab \tan^{-1} \left(\frac{a}{b} \tan \theta\right)$$
 (17)

Rewriting (16), and from a similar expression for  $\omega_B$ , we find

$$\omega_{A} = a_{0}b_{0} \left[ \tan^{-1} \left( \frac{a_{0}}{b_{0}} t_{1} \right) - \tan^{-1} \left( \frac{a_{0}}{b_{0}} t_{2} \right) \right] + a_{1}b_{1} \left[ \frac{\pi}{2} - \tan^{-1} \left( \frac{a_{1}}{b_{1}} t_{1} \right) \right]$$

$$-a_{2}b_{2} \left[ \frac{\pi}{2} - \tan^{-1} \left( \frac{a_{2}}{b_{2}} t_{2} \right) \right]$$
(18)

and

$$\omega_{\rm B} = a_{\rm o}b_{\rm o} \left[ \tan^{-1} \left( \frac{a_{\rm o}}{b_{\rm o}} \ t_3 \right) - \tan^{-1} \left( \frac{a_{\rm o}}{b_{\rm o}} \ t_4 \right) \right] + a_3b_3 \left[ \frac{\pi}{2} - \tan^{-1} \left( \frac{a_3}{b_3} \ t_3 \right) \right]$$

$$-a_4b_4 \left[ \frac{\pi}{2} - \tan^{-1} \left( \frac{a_4}{b_4} \ t_4 \right) \right] \tag{19}$$

From (6), (7), and (8) we obtain finally

$$\omega_{A} = a_{O}b_{O} \left[ \tan^{-1} \left( \frac{a_{O}}{b_{O}} \ t_{1} \right) - \tan^{-1} \left( \frac{a_{O}}{b_{O}} \ t_{2} \right) \right] + a_{1}b_{1} \left[ 0.1127\pi - \tan^{-1} \left( \frac{a_{1}}{b_{1}} \ t_{1} \right) \right]$$

$$+0.7746 \ \tan^{-1} \left( \frac{a_{1}}{b_{1}} \ t_{2} \right)$$
(20)

and

$$\omega_{\rm B} = a_{\rm O}b_{\rm O} \left[ \tan^{-1} \left( \frac{a_{\rm O}}{b_{\rm O}} \ t_{3} \right) - \tan^{-1} \left( \frac{a_{\rm O}}{b_{\rm O}} \ t_{4} \right) \right] + a_{1}b_{1} \left[ 0.1486\pi - 0.7394 \ \tan^{-1} \left( \frac{a_{1}}{b_{1}} \ t_{3} \right) \right]$$

$$+0.4422 \ \tan^{-1} \left( \frac{a_{1}}{b_{1}} \ t_{4} \right)$$
(21)

As an example, we calculate the areas for January 27, 1976. On this date the ephemeris parameters (in arc seconds) are

$$a_0 = 10.295$$
  $b_0 = 9.215$   
 $a_1 = 23.19$   $b_1 = 8.49$ 

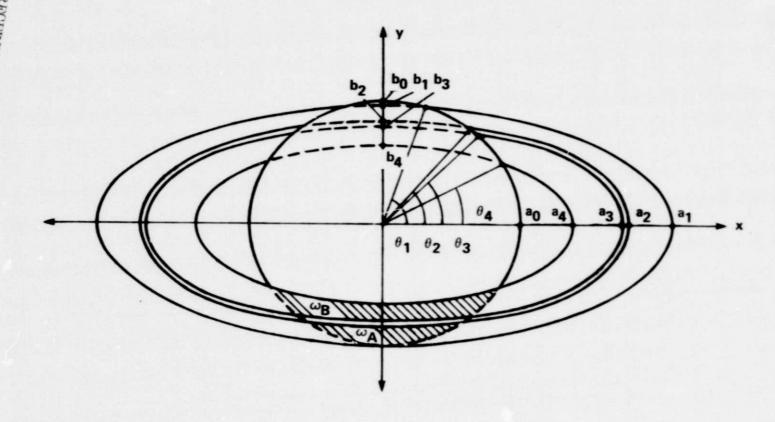
Then, from (1), (4), (9), (20), and (21) one obtains in square arc seconds the areas

$$\Omega_{\text{vd}} = 259.02$$
 $\Omega_{\text{A}} = 127.91$ 
 $\Omega_{\text{B}} = 156.25$ 
 $\omega_{\text{A}} = 11.52$ 
 $\omega_{\text{B}} = 27.49$ 
 $\Omega_{\text{t}} = 543.18$ 

#### REFERENCE

The American Ephemeris and Nautical Almanac, issued by the Nautical Almanac Office, United States Naval Observatory. U. S. Government Printing Office, Washington, 1976, p. 416.

## **SATURN GEOMETRY**



ERICKS6

Figure 1.- Notation for the calculation of the Saturn geometry. The quantities  $a_{\bf i}$  and  $b_{\bf i}$  are the semi-major and semi-minor axes of the indicated components, and the  $\theta_{\bf i}$  are the angles of intersection of the ring edges with the disk. The areas of overlap of the disk with ring A and ring B are  $\omega_A$  and  $\omega_B$ , respectively.